5.4.2 Pause

Now pause, and ask if there are some ways to apply what we have done up to now to real astrophysical objects.

- The Eddington luminosity is derived with the Thomson cross section, with the thought that it describes the smallest probability of interaction between matter and radiation. But the Klein–Nishina cross section can be even smaller, as long as the source of radiation emits at high energy. What are the consequences? If you have forgotten the definition of the Eddington luminosity, here it is:

$$L_{\rm Edd} = \frac{4\pi G M m_{\rm p} c}{\sigma_{\rm T}} = 1.3\times 10^{38}\,\frac{M}{M_\odot}\ {\rm erg\ s^{-1}} \qquad (5.15)$$

- In Nova Muscae, some years ago a (transient) annihilation line was detected, together with another feature (line–like) at 200 keV. What can this feature be?

- It seems that high energy radiation can suffer less scattering and therefore can propagate more freely through the universe. Is that true? Can you think to other processes that can kill high energy photons in space?

- Suppose to have an astrophysical source of radiation very powerful above say – 100 MeV. Assume that at some distance there is a very efficient "reflector" (i.e. free electrons) and that you can see the scattered radiation. Can you guess the spectrum you receive? Does it contain some sort of "pile–up" or not? Will this depend upon the scattering angle?

5.5 Inverse Compton scattering

When the electron is not at rest, but has an energy greater that the typical photon energy, there can be a transfer of energy from the electron to the photon. This process is called *inverse* Compton to distinguish it from the *direct* Compton scattering, in which the electron is at rest, and it is the photon to give part of its energy to the electron.

We have two regimes, that are called the *Thomson* and the *Klein–Nishina* regimes. The difference between them is the following: we go in the frame where the electron is at rest, and in that frame we calculate the energy of the incoming photon. If the latter is smaller than $m_e c^2$ we are in the Thomson regime. In this case the recoil of the electron, even if it always exists, is small, and can be neglected. In the opposite case (photon energies larger than $m_e c^2$), we are in the Klein–Nishina one, and we cannot neglect the recoil. As we shall see, in both regimes the typical photon *gain* energy, even if there will always be some arrangements of angles for which the scattered photon looses part of its energy.

5.5.1 Thomson regime

Perhaps, a better name should be "inverse Thomson" scattering, as will appear clear shortly.

5.5.2 Typical frequencies

In the frame K' comoving with the electron, the incoming photon energy is

$$x' = x\gamma(1 - \beta \cos \psi) \tag{5.16}$$

where ψ is the angle between the electron velocity and the photon direction (see Fig 5.5).

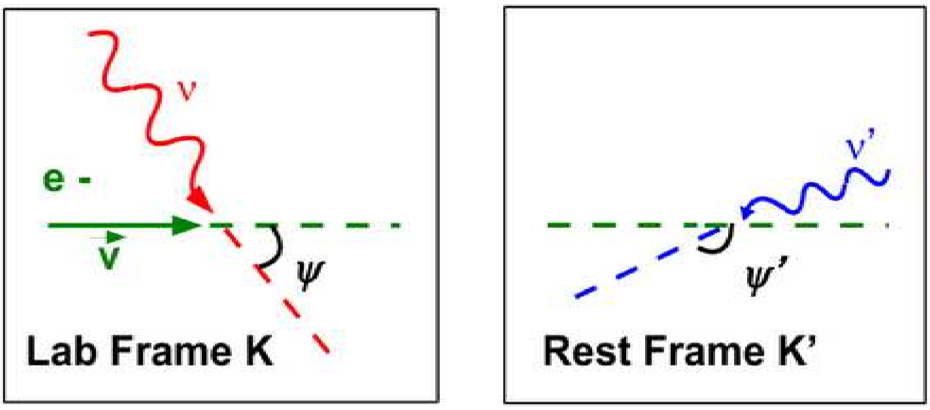

Figure 5.5: In the lab frame an electron is moving with velocity $\mathbf{v}$. Its velocity makes an angle ψ with an incoming photon of frequency ν. In the frame where the electron is at rest, the photon is coming from the front, with frequency ν', making an angle ψ' with the direction of the velocity.

At first sight this is different from $x' = x\delta$ derived in Chapter 3. But notice that i) in this case the angle ψ is measured in the lab frame; ii) it is not the same angle going into the definition of δ (i.e. in δ we use the angle between the line of sight and the velocity of the emitter, i.e. $\theta' = \pi - \psi'$).

Going to the rest frame of the electrons we should use (recalling Eq. 3.16 for the transformation of angles):

$$\cos\psi \;=\; \frac{\beta+\cos\psi'}{1+\beta\cos\psi'} \tag{5.17}$$

Substituting this into equation 5.16 we have

$$x' \;=\; \frac{x}{\gamma(1+\beta\cos\psi')} \tag{5.18}$$

Finally, consider that $\cos\theta' = \cos(\pi-\psi') = -\cos\psi'$, validating $x' = x\delta$.

If $x' \ll 1$, we are in the Thomson regime. In the rest frame of the electron the scattered photon will have the same energy x'_1 as before the scattering, independent of the scattering angle. Then

$$x'_1 = x' \tag{5.19}$$

This photon will be scattered at an angle ψ'_1 with respect to the electron velocity. The pattern of the scattered radiation will follow the pattern of the cross section (i.e. a peanut). *Think to the scattering in the comoving frame as a re-isotropization process: even if the incoming photons are all coming from the same direction, after the scattering they are distributed quasi–isotropically.* Going back to K the observer sees

$$x_1 \;=\; x'_1\gamma(1+\beta\cos\psi'_1) \tag{5.20}$$

Recalling again Eq. 3.16, for the transformation of angles:

$$\cos\psi'_1 \;=\; \frac{\cos\psi_1-\beta}{1-\beta\cos\psi_1} \tag{5.21}$$

we arrive to the final formula:

$$x_1 \;=\; x\,\frac{1-\beta\cos\psi}{1-\beta\cos\psi_1} \tag{5.22}$$

Now all quantities are calculated in the lab–frame.

Let us see the minimum and maximum energies. The maximum is when $\psi=\pi$ (head on collision), and when $\psi_1 = 0$ (the photon is scattered along the electron velocity vector). In these head–on collisions:

$$x_1 \;=\; x\,\frac{1+\beta}{1-\beta} \;=\; \gamma^2(1+\beta)^2 x \;\to\; 4\gamma^2 x; \quad \mathrm{head-on} \tag{5.23}$$

where the last step is valid if $\gamma \gg 1$. The other extreme is for $\psi_1=\pi$ and $\psi = 0$. In this case the incoming photon "comes from behind" and bounces back. In these "tail–on" collisions:

$$x_1 \;=\; x\,\frac{1-\beta}{1+\beta} \;=\; \frac{x}{\gamma^2(1+\beta)^2} \;\to\; \frac{x}{4\gamma^2}; \quad \mathrm{tail-on} \tag{5.24}$$

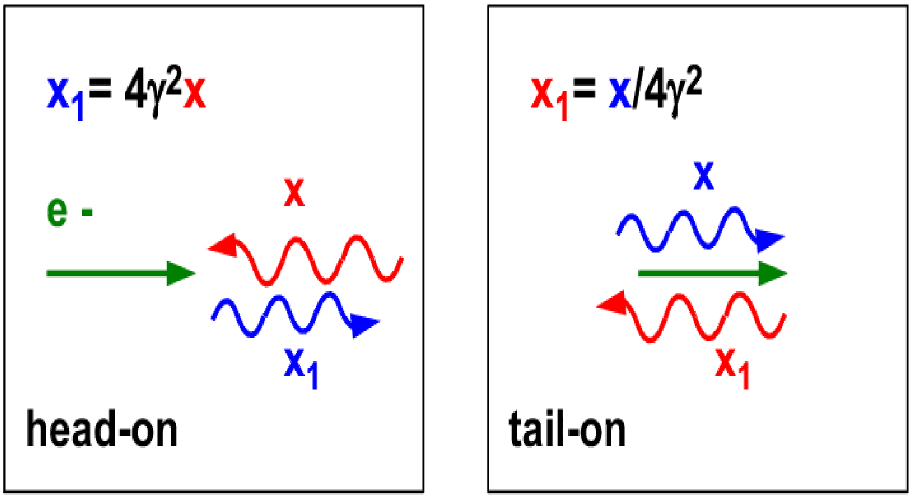

Figure 5.6: Maximum and minimum scattered frequencies. The maximum occurs for head-on collisions, the minimum for tail–on ones. These two frequencies are one the inverse of the other.

where again the last step is valid if $\gamma \gg 1$. Another typical angle is $\sin\psi_1 = 1/\gamma$, corresponding to $\cos\psi_1 = \beta$. This corresponds to the aperture angle of the beaming cone. For this angle:

$$x_1 = \frac{1-\beta\cos\psi}{1-\beta^2}x = \gamma^2(1-\beta\cos\psi)x; \quad \text{beaming cone} \tag{5.25}$$

which becomes $x_1 = x/(1+\beta)$ for $\psi = 0$, $x_1 = \gamma^2 x$ for $\psi = \pi/2$ and $x_1 = \gamma^2(1+\beta)x$ for $\psi = \pi$.

For an isotropic distribution of incident photons and for $\gamma \gg 1$ the average photon energy after scattering is (see Eq. 5.47):

$$\langle x_1 \rangle = \frac{4}{3}\gamma^2 x \tag{5.26}$$

Total loss rate

We can simply calculate the rate of scatterings per electron considering all quantities in the lab–frame. Let $n(\epsilon)$ be the density of photons of energy $\epsilon = h\nu$, v the electron velocity and ψ the angle between the electron velocity and the incoming photon. For mono–directional photon distributions, we have:

$$\frac{dN}{dt} = \int \sigma_{\rm T} v_{\rm rel} n(\epsilon) d\epsilon \tag{5.27}$$

$v_{\rm rel} = c - v\cos\psi$ is the relative velocity between the electron and the incoming photons. We then have

$$\frac{dN}{dt} = \int \sigma_{\rm T} c(1-\beta\cos\psi)n(\epsilon)d\epsilon \tag{5.28}$$

Note that the rate of scatterings in the lab frame, when the electron and/or photon are anisotropically distributed, can be described by an effective cross section $\sigma_{\rm eff} \equiv \int \sigma_{\rm T}(1-\beta\cos\psi)d\Omega/4\pi$. For photons and electrons moving in the same direction the scattering rate (hence, the effective optical depth) can be greatly reduced.

The power contained in the scattered radiation is then

$$\frac{dE_\gamma}{dt} = \frac{\epsilon_1 dN}{dt} = \sigma_{\rm T} c \int \frac{(1-\beta\cos\psi)^2}{1-\beta\cos\psi_1}\epsilon n(\epsilon)d\epsilon \tag{5.29}$$

Independently of the incoming photon angular distribution, the average value of $1-\beta\cos\psi_1$ can be calculated recalling that, in the rest frame of the electron, the scattering has a backward–forward symmetry, and therefore $\langle\cos\psi_1'\rangle = \pi/2$. The average value of $\cos\psi_1$ is then β, leading to $\langle 1-\beta\cos\psi_1\rangle = 1/\gamma^2$. We therefore obtain

$$\frac{dE_\gamma}{dt} = \sigma_{\rm T} c\gamma^2 \int (1-\beta\cos\psi)^2 \epsilon n(\epsilon)d\epsilon \tag{5.30}$$

If the incoming photons are isotropically distributed, we can average out $(1-\beta\cos\psi)^2$ over the solid angle, obtaining $1+\beta^2/3$. The power produced is then

$$\frac{dE_\gamma}{dt} = \sigma_{\rm T} c\gamma^2 \left(1+\frac{\beta^2}{3}\right) U_{\rm r} \tag{5.31}$$

where

$$U_{\rm r} = \int \epsilon n(\epsilon)d\epsilon \tag{5.32}$$

is the energy density of the radiation before scattering. This is the power contained in the scattered radiation. To calculate the energy loss rate of the electron, we have to subtract the initial power of the radiation eventually scattered

$$P_{\rm c}(\gamma) \equiv \frac{dE_e}{dt} = \frac{dE_\gamma}{dt} - \sigma_{\rm T} c U_{\rm r} = \frac{4}{3}\sigma_{\rm T} c\gamma^2\beta^2 U_{\rm r} \tag{5.33}$$

A simple way to remember Eq. 5.33 is:

$$\begin{aligned} P_{\rm c}(\gamma) &= \left(\frac{\#\text{ of collisions}}{\text{sec}}\right)(\text{average phot. energy after scatt.}) \\ &= \left(\sigma_{\rm T} c\frac{U_{\rm r}}{\langle h\nu\rangle}\right)\left(\frac{4}{3}\langle h\nu\rangle\gamma^2\right) \end{aligned} \tag{5.34}$$

Note the similarity with the synchrotron energy loss. The two energy loss rates are identical, once the radiation energy density is replaced by the magnetic energy density U_B. Therefore, if relativistic electrons are in a region with some radiation and magnetic energy densities, they will emit by both the synchrotron and the Inverse Compton scattering processes. The ratio of the two luminosities will be

$$\frac{L_{\rm syn}}{L_{\rm IC}} = \frac{P_{\rm syn}}{P_{\rm c}} = \frac{U_{\rm B}}{U_{\rm r}} \tag{5.35}$$

where we have set $dE_{\rm IC}/dt = dE_e/dt$. This is true unless one of the two processes is inhibited for some reason. For instance:

- At (relatively) low energies, electrons could emit *and absorb* synchrotron radiation, so the synchrotron cooling is compensated by the heating due to the absorption process.
- At high energies, electrons could scatter in the Klein–Nishina regime: in this case, since the cross section is smaller, they will do less scatterings, and cool less.

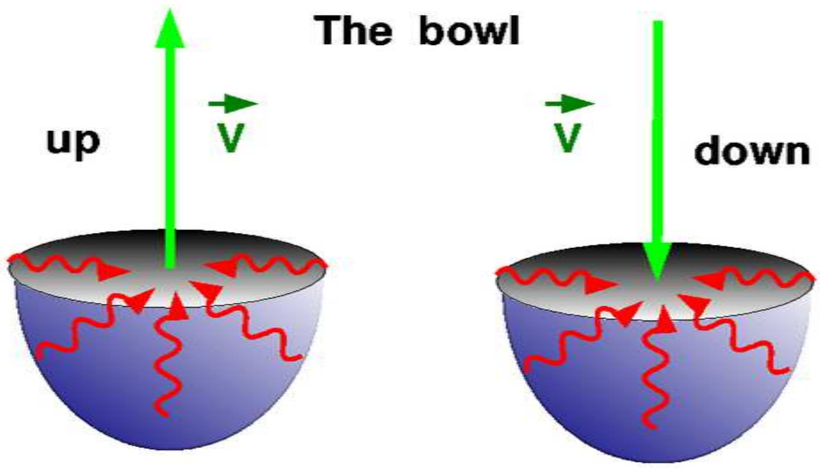

Figure 5.7: In the center of a semi–sphere (the "bowl") we have relativistic electrons going down and going up, all with the same γ. Since the seed photon distribution is anisotropic, so is the scattered radiation and power. The losses of the electron going down are 7 times larger than those of the electron going up (if $\gamma \gg 1$). Since almost all the radiation is produced along the velocity vector of the electrons, also the downward radiation is 7 times more powerful than the upward radiation.

But let us go back to Eq. 5.30, that is the starting point when dealing with *anisotropic* seed photon distributions. Think for instance to an accretion disk as the producer of the seed photons for scattering, and some cloud of relativistic electrons above the disk. If the cloud is not that distant, and it is small with respect to the disk size, then this case is completely equal to the case of having a little cloud of relativistic electrons located at the center of a semi–sphere. That is, we have the "bowl" case illustrated in Fig 5.7. Just for fun, let us calculate the total power emitted by an electron going "up" and by its brother (i.e. it has the same γ) going down. Using Eq. 5.30 we have:

$$\frac{P_{\rm down}}{P_{\rm up}} = \frac{\int_{-1}^{0}(1-\beta\mu)^2 d\mu}{\int_0^1 (1-\beta\mu)^2 d\mu} = \frac{1+\beta+\beta^2/3}{1-\beta+\beta^2/3} \to 7 \tag{5.36}$$

where $\mu \equiv \cos\psi$ and the last step assumes $\beta \to 1$. Since almost all the radiation is produced along the velocity vector of the electrons, also the downward radiation is more powerful than the upward radiation (i.e. 7 times more powerful for $\gamma \gg 1$). What happens if the cloud of electrons is located at some height above the bowl? Will the $P_{\rm down}/P_{\rm up}$ be more or less?

5.5.3 Cooling time and compactness

The cooling time due to the inverse Compton process is

$$t_{\rm IC} = \frac{E}{dE_e/dt} = \frac{3\gamma m_e c^2}{4\sigma_{\rm T} c\gamma^2\beta^2 U_{\rm r}} \sim \frac{3m_e c^2}{4\sigma_{\rm T} c\gamma U_{\rm r}}; \qquad \gamma\epsilon \ll m_{\rm e}c^2 \tag{5.37}$$

This equation offers the opportunity to introduce an important quantity, namely *the compactness* of an astrophysical source, that is essentially the luminosity L over the size R ratio. Consider in fact how $U_{\rm r}$ and L are related:

$$U_{\rm r} = \frac{L}{4\pi R^2 c} \tag{5.38}$$

Although this relation is almost universally used, there are subtleties. It is surely valid if we measured $U_{\rm r}$ *outside* the source, at a distance R from its center. In this case $4\pi R^2 c$ is simply the volume of the shell crossed by the source radiation in one second. But if we are *inside* an homogeneous, spherical transparent source, a better way to calculate $U_{\rm r}$ is to think to the average time needed to the typical photon to exit the source. This is $t_{\rm esc} = 3R/(4c)$. It is less than R/c because the typical photon is not born at the center (there is more volume close to the surface). If $V = (4\pi/3)R^3$ is the volume, we can write:

$$U_{\rm r} = \frac{L}{V} t_{\rm esc} = \frac{3L}{4\pi R^3}\frac{3R}{4c} = \frac{9L}{16\pi R^2 c} \tag{5.39}$$

This is greater than Eq. 5.38 by a factor 9/4. Anyway, let us be conventional and insert Eq. 5.38 in Eq. 5.37:

$$t_{\rm IC} = \frac{3\pi m_e c^2 R^2}{\sigma_{\rm T}\gamma L} \to \frac{t_{\rm IC}}{R/c} = \frac{3\pi}{\gamma}\frac{m_e c^3 R}{\sigma_{\rm T} L} \equiv \frac{3\pi}{\gamma}\frac{1}{\ell} \tag{5.40}$$

where the dimensionless compactness ℓ is defined as

$$\ell = \frac{\sigma_{\rm T} L}{m_{\rm e} c^3 R} \tag{5.41}$$

For ℓ close or larger than unity, we have that even low energy electrons cool by the Inverse Compton process in less than a light crossing time R/c.

There is another reason why ℓ is important, related to the fact that it directly measures the optical depth (hence the probability to occur) of the photon–photon collisions that lead to the creation of electron–positron pairs. The compactness is one of the most important physical parameters when studying high energy compact sources (X–ray binaries, AGNs and Gamma Ray Bursts).

5.5.4 Single particle spectrum

As we did for the synchrotron spectrum, we will not repeat the exact derivation of the single particle spectrum, but we try to explain why the typical frequency of the scattered photon is a factor γ^2 larger than the frequency of the incoming photon. Here are the steps to consider:

1. Assume that the relativistic electron travels in a region where there is a radiation energy density $U_{\rm r}$ made by photons which we will take, for simplicity, monochromatic, therefore all having a dimensionless frequency $x = h\nu/m_{\rm e}c^2$.

2. In the frame where the electron is at rest, half of the photons appear to come from the front, within an angle $1/\gamma$.

3. The typical frequency of these photons is $x' \sim \gamma x$ (it is twice that for photons coming exactly head on).

4. Assuming that we are in the Thomson regime means that i) $x' < 1$; ii) the cross section is the Thomson one; iii) the frequency of the scattered photon is the same of the incoming one, i.e. $x'_1 = x' \sim \gamma x$, and iv) the pattern of the scattered photons follows the angular dependence of the cross section, therefore the "peanut".

5. Independently of the initial photon direction, and therefore independently of the frequencies seen by the electrons, all photons after scatterings are isotropized. This means that all observers (at any angle ψ'_1 in this frame see the same spectrum, and the same typical frequency. Half of the photons are in the semi-sphere with $\psi'_1 \leq \pi/2$.

6. Now we go back to the lab–frame. Those photons that had $\psi'_1 \leq \pi/2$ now have $\psi_1 \leq 1/\gamma$. Their typical frequency if another factor γ greater than what they had in the rest frame, therefore

$$x_1 \sim \gamma^2 x \tag{5.42}$$

This is the typical Inverse Compton frequency.

The exact derivation can be found e.g. in Rybicki & Lightman (1979) and in Blumenthal & Gould (1970). We report here the final result, valid for a monochromatic and isotropic seed photons distribution, characterized by a specific intensity

$$\frac{I(x)}{x} = \frac{I_0}{x}\delta(x - x_0) \tag{5.43}$$

Note that $I(x)/x$ is the analog of the normal intensity, but it is associated with the number of photons. If we have n electrons per cubic centimeter we have:

$$\epsilon_{\rm IC}(x_1) = \frac{\sigma_{\rm T} n I_0 (1+\beta)}{4\gamma^2\beta^2 x_0} F_{\rm IC}(x_1) \tag{5.44}$$

The function $F_{\rm IC}$ contains all the frequency dependence:

$$\begin{aligned} F_{\rm IC}(x_1) &= \frac{x_1}{x_0}\left[\frac{x_1}{x_0} - \frac{1}{(1+\beta)^2\gamma^2}\right]; \quad \frac{1}{(1+\beta)^2\gamma^2} < \frac{x_1}{x_0} < 1 \\ F_{\rm IC}(x_1) &= \frac{x_1}{x_0}\left[1 - \frac{x_1}{x_0}\frac{1}{(1+\beta)^2\gamma^2}\right]; \quad 1 < \frac{x_1}{x_0} < (1+\beta)^2\gamma^2 \end{aligned} \tag{5.45}$$

The first line corresponds to *downscattering*: the scattered photon has *less* energy than the incoming one. Note that in this case $F_{\rm IC}(x_1) \propto x_1^2$. The second line corresponds to *upscattering*: in this case $F_{\rm IC}(x_1) \propto x_1$ except for frequencies close to the maximum ones. The function $F_{\rm IC}(x_1)$ is shown in Fig. 5.8 for different values of γ. The figure shows also the spectrum of the photons contained in the beaming cone $1/\gamma$: the corresponding power is always 75% of the total.

The average frequency of $F_{\rm IC}(x_1)$ is

$$\langle x_1\rangle = 2\gamma^2 x_0; \quad \text{energy spectrum} \tag{5.46}$$

This is the average frequency *of the energy spectrum*. We sometimes want to know the average energy *of the photons*, i.e. we have to calculate the average frequency of *the photon spectrum* $F_{\rm IC}(x_1)/x_1$. This is:

$$\langle x_1\rangle = \frac{4}{3}\gamma^2 x_0; \quad \text{photon spectrum} \tag{5.47}$$

5.6 Emission from many electrons

We have seen that the emission spectrum from a single particle is peaked, and the typical frequency is boosted by a factor γ^2. This is equal to the synchrotron case. Therefore we can derive the Inverse Compton emissivity as we did for the synchrotron one. Again, assume a power–law energy distribution for the relativistic electrons:

$$N(\gamma) = K\gamma^{-p} = N(E)\frac{dE}{d\gamma}; \quad \gamma_{\rm min} < \gamma < \gamma_{\rm max} \tag{5.48}$$

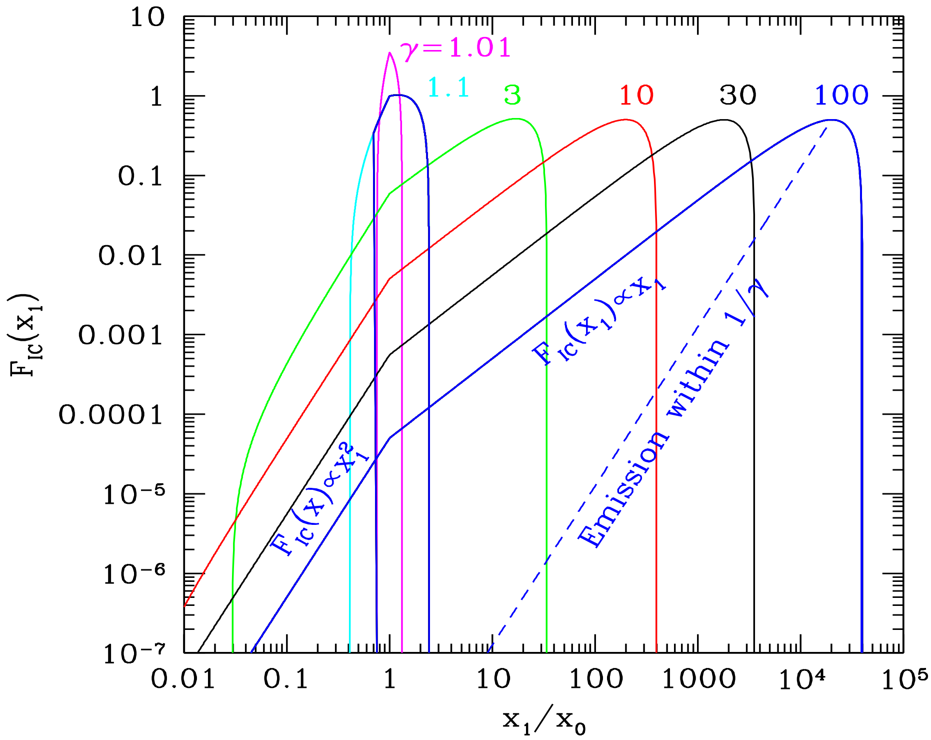

Figure 5.8: Spectrum emitted by the Inverse Compton process by electrons of different γ (as labeled) scattering an isotropic monochromatic radiation field of dimensionless frequency x_0. The dashed line corresponds to the spectrum emitted within the $1/\gamma$ beming cone: it always contains the 75% of the total power, for any γ. For $x_1 < x_0$ we have *downscattering*, i.e. the photons loose energy in the process. Note also the power law segments arising when $\gamma \gg 1$: $F_{\rm IC}(x_1) \propto x_1^2$ for the downscattering tail, and $F_{IC}(x_1) \propto x_1$ for the upscattering segment.

and assume that it describes an isotropic distribution of electrons. For simplicity, let us assume that the seed photons are isotropic and monochromatic, with frequency ν_0 (we now pass to real frequencies, since we are getting closer to the real world..). Since there is a strong link between the scattered frequency $\nu_{\rm c}$ and the electron energy that produced it, we can set:

$$\nu_{\rm c} = \frac{4}{3}\gamma^2\nu_0 \to \gamma = \left(\frac{3\nu_{\rm c}}{4\nu_0}\right)^{1/2} \to \left|\frac{d\gamma}{d\nu}\right| = \frac{\nu_{\rm c}^{-1/2}}{2}\left(\frac{3}{4\nu_o}\right)^{1/2} \qquad (5.49)$$

Now, repeating the argument we used for synchrotron emission, we can state that the power lost by the electron of energy $\gamma m_{\rm e}c^2$ within $m_{\rm e}c^2 d\gamma$ goes into the radiation of frequency ν within $d\nu$. Since we will derive an emissivity (i.e. erg cm^{-3} s^{-1} sterad^{-1} Hz^{-1}) we must remember the 4π term (if the emission is isotropic). We can set:

$$\epsilon_c(\nu_c)d\nu_c = \frac{1}{4\pi} m_e c^2 P_c(\gamma)N(\gamma)d\gamma \tag{5.50}$$

This leads to:

$$\epsilon_c(\nu_c) = \frac{1}{4\pi}\frac{(4/3)^\alpha}{2}\sigma_T c K \frac{U_r}{\nu_0}\left(\frac{\nu_c}{\nu_0}\right)^{-\alpha} \tag{5.51}$$

Again, a power law, as in the case of synchrotron emission by a power law energy distribution. Again the same link between α and p:

$$\alpha = \frac{p-1}{2} \tag{5.52}$$

Of course, this is not a coincidence: it is because both the Inverse Compton and the synchrotron single electron spectra are peaked at a typical frequency that is a factor γ^2 greater than the starting one.

Eq. 5.51 becomes a little more clear if

- we express $\epsilon_c(\nu_c)$ as a function of the photon energy $h\nu_c$. Therefore $\epsilon_c(h\nu_c) = \epsilon_c(\nu_c)/h$;
- we multiply and divide by the source radius R;
- we consider a proxy for the scattering optical depth of the relativistic electrons setting $\tau_c \equiv \sigma_T K R$.

Then we obtain:

$$\epsilon_c(h\nu_c) = \frac{1}{4\pi}\frac{(4/3)^\alpha}{2}\frac{\tau_c}{R/c}\frac{U_r}{h\nu_0}\left(\frac{\nu_c}{\nu_0}\right)^{-\alpha} \tag{5.53}$$

In this way: τ_c (for $\tau_c < 1$) is the fraction of the seed photons $U_r/h\nu_0$ undergoing scattering in a time R/c, and $\nu_c/\nu_0 \sim \gamma^2$ is the average gain in energy of the scattered photons.

5.6.1 Non monochromatic seed photons

It is time to consider the more realistic case in which the seed photons are not monochromatic, but are distributed in frequency. This means that we have to integrate Eq. 5.51 over the incoming photon frequencies. For clarity, let us drop the subscript 0 in ν_0. We have

$$\epsilon_c(\nu_c) = \frac{1}{4\pi}\frac{(4/3)^\alpha}{2}\frac{\tau_c}{R/c}\nu_c^{-\alpha}\int_{\nu_{\min}}^{\nu_{\max}}\frac{U_r(\nu)}{\nu}\nu^\alpha d\nu \tag{5.54}$$

where $U_r(\nu)$ [erg cm^{-3} Hz^{-1}] is the specific radiation energy density at the frequency ν. The only difficulty of this integral is to find the correct limit of the integration, that, in general, *depend on* ν_c. Note also another interesting thing. We have just derived that if the same electron population produces Inverse Compton and synchrotron emission, than the slopes of the two spectra are the same. Therefore, when $U_{\rm r}(\nu)$ is made by synchrotron photons, then $U_{\rm r}(\nu) \propto \nu^{-\alpha}$. The result of the integral, in this case, will be $\ln(\nu_{max}/\nu_{min})$.

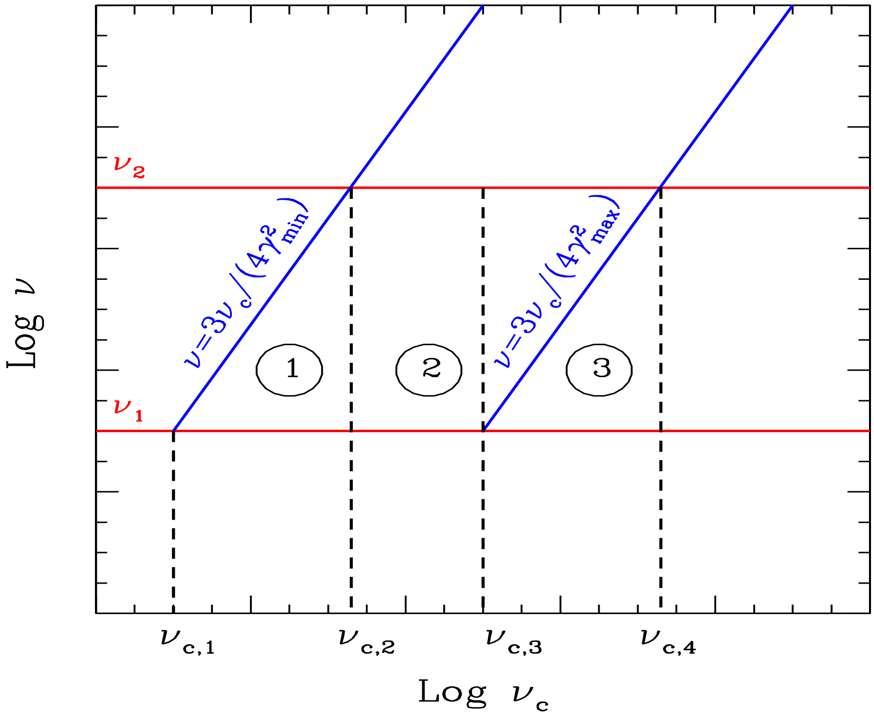

Figure 5.9: The ν–$\nu_{\rm c}$ plane. The two diagonal lines delimit the regions of the seed photons that can be used to give a given frequency $\nu_{\rm c}$.

Fig. 5.9 helps to understand what are the right ν_{max} and ν_{min} to use. On the y–axis we have the frequencies of the seed photon distribution, which extend between ν_1 and ν_2. On the x–axis we have the scattered frequencies, which extend between $\nu_{\rm c,1} = (4/3)\gamma^2_{\rm min}\nu_1$ and $\nu_{\rm c,4} = (4/3)\gamma^2_{\rm max}\nu_1$. The diagonal lines are the functions

$$\nu = \frac{3\nu_{\rm c}}{4\gamma^2_{\rm min}} \qquad \nu = \frac{3\nu_{\rm c}}{4\gamma^2_{\rm max}} \tag{5.55}$$

that tell us what are the appropriate ν that can give $\nu_{\rm c}$ once we change γ.

There are three zones:

1. In zone (1), between $\nu_{\rm c,1}$ and $\nu_{\rm c,2} = (4/3)\gamma^2_{\rm min}\nu_2$ the appropriate limits of integration are:

$$\nu_{\rm min} = \nu_1 \qquad \nu_{\rm max} = \frac{3\nu_{\rm c}}{4\gamma^2_{\rm min}} \tag{5.56}$$

2. In zone (2), between $\nu_{c,2}$ and $\nu_{c,3} = (4/3)\gamma^2_{max}\nu_1$ the limits are:

$$\nu_{min} = \nu_1 \qquad \nu_{max} = \nu_2 \tag{5.57}$$

3. In zone (3), between $\nu_{c,3}$ and $\nu_{c,4} = (4/3)\gamma^2_{max}\nu_2$ the limits are:

$$\nu_{min} = \frac{3\nu_c}{4\gamma^2_{max}} \qquad \nu_{max} = \nu_2 \tag{5.58}$$

We see that only in zone (2) the limits of integration coincide with the extension in frequency of the seed photon distribution, and are therefore constant. Therefore $\epsilon_c(\nu_c)$ will be a power law of slope α only in the corresponding frequency limits. Note also that for a broad range in $[\nu_1;\nu_2]$ or a narrow range in $[\gamma_{min};\gamma_{max}]$ we do not have a power law, since there is no ν_c for which the limits of integrations are both constants.

5.7 Thermal Comptonization

With this term we mean the process of *multiple* scattering of a photon due to a *thermal* or *quasi–thermal* distribution of electrons. By *quasi–thermal* we mean a particle distribution that is peaked, even if it is not a perfect Maxwellian. Since the resulting spectrum, by definition, is due to the superposition of many spectra, each corresponding to a single scattering, the details of the particle distribution will be lost in the final spectrum, as long as the distribution is peaked. The "bible" for an extensive discussion about this process is Pozdnyakov, Sobol & Sunyaev (1983).

There is one fundamental parameter measuring the importance of the Inverse Compton process in general, and of multiple scatterings in particular: *the Comptonization parameter*, usually denoted with the letter y. Its definition is:

$$y = [\text{average \# of scatt.}] \times [\text{average fractional energy gain for scatt.}] \tag{5.59}$$

If $y > 1$ the Comptonization process is important, because the Comptonized spectrum has more energy than the spectrum of the seed photons.

5.7.1 Average number of scatterings

This can be calculated thinking that the photon, before leaving the source, experience a sort of random walk inside the source. Let us call

$$\tau_T = \sigma_T n R \tag{5.60}$$

the Thomson scattering optical depth, where n is the electron density and R the size of the source. When $\tau_T < 1$ most of the photons leave the source directly, without any scattering. When $\tau_T > 1$ then the mean free path is $d = R/\tau_T$ and the photon will experience, on average, τ_T^2 scatterings before leaving the source. Therefore the total path travel by the photon, from the time of its birth to the time it leaves the source is: the photon is born, is

$$c\Delta t = \tau_T^2 \frac{R}{\tau_T} = \tau_T R \tag{5.61}$$

and Δt is the corresponding elapsed time.

5.7.2 Average gain per scattering

Relativistic case

If the scattering electrons are relativistic, we have already seen that the photon energy is amplified by the factor $(4/3)\gamma^2$ (on average). Therefore the problem is to find what is $\langle\gamma^2\rangle$ in the case of a relativistic Maxwellian, that has the form

$$N(\gamma) \propto \gamma^2 e^{-\gamma/\Theta}; \quad \Theta \equiv \frac{kT}{m_e c^2} \tag{5.62}$$

Setting $x_0 = h\nu_0/(m_e c^2)$ we have that the average energy of the photon of initial frequency x_0 after a single scattering with electron belonging to this Maxwellian is:

$$\begin{aligned}\langle x_1\rangle &= \frac{4}{3}\langle\gamma^2\rangle = \frac{4}{3}x_0 \frac{\int_1^\infty \gamma^2\gamma^2 e^{-\gamma/\Theta} d\gamma}{\int_1^\infty \gamma^2 e^{-\gamma/\Theta} d\gamma} \\ &= \frac{4}{3}x_0\Theta^2\frac{\Gamma(5)}{\Gamma(3)} \\ &= \frac{4}{3}x_0\frac{4!}{2!} = 16\Theta^2 x_0\end{aligned} \tag{5.63}$$

Non relativistic case

In this case the average gain is proportional to the electron energy, not to its square. The derivation is not immediate, but we must use a trick. Also, we have to account that in any Maxwellian, but especially when the temperature is not large, there will be electrons that have *less* energy than the incoming photons. In this case it is the photon to give energy to the electron: correspondingly, the scattered photon will have less energy than the incoming one. Averaging out over a Maxwellian distribution, we will have:

$$\frac{\Delta x}{x} = \frac{x_1 - x_0}{x_0} = \alpha\Theta - x \tag{5.64}$$

Where $\alpha\Theta$ is what the photon gains and the $-x$ term corresponds to the downscattering of the photon (i.e. direct Compton). We do not know yet the value for the constant α. To determine it we use the following argument.

We know (from general and robust arguments) what happens when photons and electrons are in equilibrium under the only process of scattering, and neglecting absorption (i.e. when the *number of photon is conserved*). What happens is that the photons follow the so–called *Wien distribution* given by:

$$F_{\rm W}(x) \,\propto\, x^3 e^{-x/\Theta} \to N_{\rm W}(x) \,=\, \frac{F_{\rm W}(x)}{x} \,\propto x^2 e^{-x/\Theta} \tag{5.65}$$

where F correspond to the radiation spectrum, N to the photon spectrum, and Θ is the dimensionless electron temperature. When a Wien distribution is established we must have $\langle \Delta x\rangle = 0$, since we are at equilibrium, So we require that, on average, gains equal losses:

$$\langle \Delta x\rangle \,= 0 \,\to\, \alpha\Theta\langle x\rangle - \langle x^2\rangle \,=\, 0 \tag{5.66}$$

Calculating $\langle x\rangle$ and $\langle x^2\rangle$ for a photon Wien distribution, we have:

$$\begin{aligned} \langle x\rangle \quad &= \quad \frac{\int_0^\infty x^3 e^{-x/\Theta} dx}{\int_0^\infty x^2 e^{-x/\Theta} dx} \,=\, \frac{\Gamma(4)}{\Gamma(3)}\,\Theta \,=\, \frac{3!}{2!}\,\Theta \,=\, 3\,\Theta \\ \langle x^2\rangle \quad &= \quad \frac{\int_0^\infty x^4 e^{-x/\Theta} dx}{\int_0^\infty x^2 e^{-x/\Theta} dx} \,=\, \frac{\Gamma(5)}{\Gamma(3)}\,\Theta^2 \,=\, \frac{4!}{2!}\,\Theta^2 \,=\, 12\Theta^2 \end{aligned} \tag{5.67}$$

This implies that $\alpha = 4$ not only at equilibrium, but always, and we finally have

$$\frac{\Delta x}{x} \,=\, 4\Theta - x \tag{5.68}$$

Combining the relativistic and the non relativistic cases, we have an expression valid for all temperatures:

$$\frac{\Delta x}{x} \,=\, 16\Theta^2 + 4\Theta - x \tag{5.69}$$

going back to the y parameter we can write:

$$y \,=\, \max(\tau_{\rm T}, \tau_{\rm T}^2) \times [16\Theta^2 + 4\Theta - x] \tag{5.70}$$

Going to the differential form, and neglecting downscattering, we have

$$\frac{dx}{x} \,=\, [16\Theta^2 + 4\Theta]\, dK \,\to x_{\rm f} \,=\, x_0\, e^{(16\Theta^2+4\Theta)K} \,\to x_{\rm f} \,=\, x_0\, e^y \tag{5.71}$$

where now K is the number of scatterings. If we subtract the initial photon energy, and consider that the above equation is valid for all the x_0 of the initial seed photon distribution, of luminosity L_0, we have

$$\frac{L_{\rm f}}{L_0} \,=\, e^y - 1 \tag{5.72}$$

Then the importance of y is self evident, and also the fact that it marks the importance of the Comptonization process when it is larger than 1.

# scatt.	Fraction of escaping photons	$\langle x \rangle$
0	$e^{-\tau_T} \to 1 - \tau_T$	x_0
1	$\sim \tau_T$	$x_0 A$
2	$\sim \tau_T^2$	$x_0 A^2$
3	$\sim \tau_T^3$	$x_0 A^3$
4	$\sim \tau_T^4$	$x_0 A^4$
....	......	....
n	$\sim \tau_T^n$	$x_0 A^n$

Table 5.1: When $\tau_T < 1$, a fraction $e^{-\tau_T}$ of the seed photons escape without doing any scattering, and a fraction $1 - e^{-\tau_T} \to \tau$ undergoes at least one scattering. We can then repeat these fractions for all scattering orders. Even if a tiny fraction of photons does several scatterings, they can carry a lot of energy.

5.7.3 Comptonization spectra: basics

We will illustrate why even a thermal (Maxwellian) distribution of electrons can produce a power law spectrum. The basic reason is that the total produced spectrum is the superposition of many orders of Compton scattering spectra: when they are not too much separated in frequency (i.e. for not too large temperatures) the sum is a smooth power law. We can distinguish 4 regimes, according to the values of τ_T and y. As usual, we set $x \equiv h\nu/(m_e c^2)$ and $\Theta \equiv kT/(m_e c^2)$.

The case $\tau_T < 1$

Neglect downscattering for simplicity. The fractional energy gain is $\Delta x/x = 16\Theta^2 + 4\Theta$, so the amplification A of the photon frequency at each scattering is

$$A \equiv \frac{x_1}{x} = 16\Theta^2 + 4\Theta + 1 \sim \frac{y}{\tau_T} \tag{5.73}$$

We can then construct Table 5.1.

A look to Fig. 5.10 should convince you that the sum of all the scattering orders gives a power law, and should also make clear how to find the spectral slope. Remember that we are in a log–log plot, so the spectral index is simply $\Delta y/\Delta x$. We can find it considering two successive scattering orders: the typical (logarithm of) frequency is enhanced by $\log A$, and the fraction of photons doing the scattering is $-\log \tau_T$. Remember also that we use $F(x) \propto x^{-\alpha}$ as the definition of energy spectral index.

Therefore

$$\alpha = -\frac{\log \tau_T}{\log A} \sim -\frac{\log \tau_T}{\log y - \log \tau_T} \tag{5.74}$$

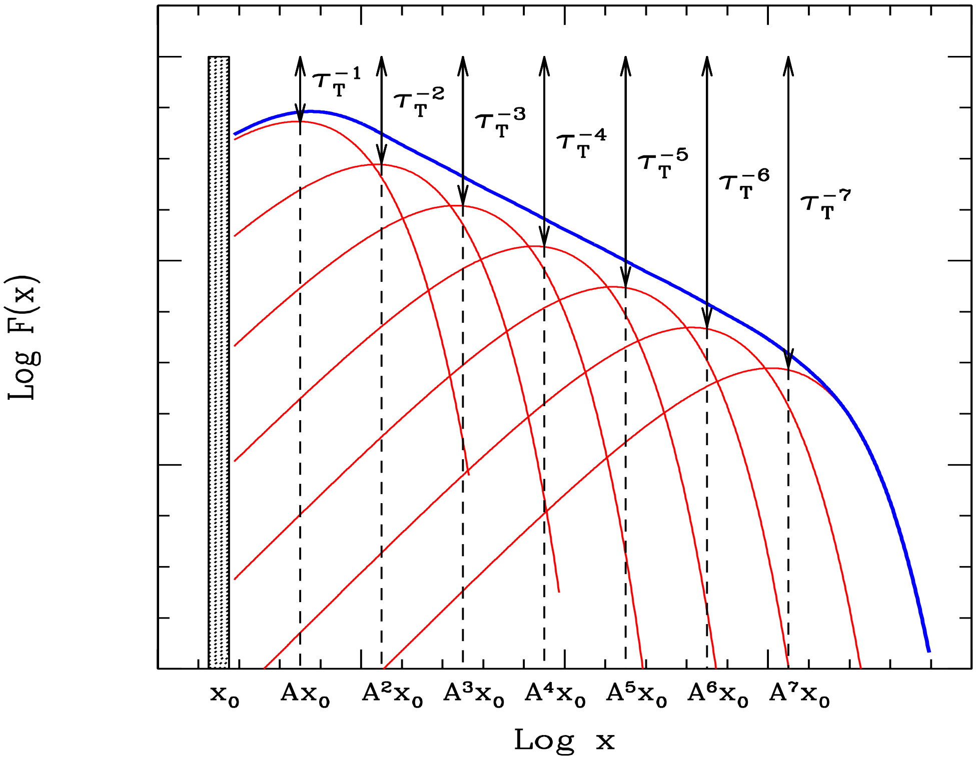

Figure 5.10: Multiple Compton scatterings when $\tau_T < 1$. A fraction τ_T of the photons of the previous scattering order undergoes another scattering, and amplifying the frequency by the gain factor A, until the typical photon frequency equals the electron temperature Θ. Then further scatterings leave the photon frequency unchanged.

When $y \sim 1$, its logarithm is close to zero, and we have $\alpha \sim 1$. When $y > 1$, then $\alpha < 1$ (i.e. *flat, or hard*), and vice–versa, when $y < 1$, then its logarithm will be negative, as the logarithm of τ_T, and $\alpha > 1$ (i.e. *steep, or soft*).

Attention! when $\tau_T \ll 1$ and A is large (i.e. big frequency jumps between one scattering and the next), then the superposition of all scattering orders (by the way, there are fewer, in this case) will not guarantee a perfect power–law. In the total spectrum we can see the "bumps" corresponding to individual scattering orders.

The case $\tau_T \gtrsim 1$

This is the most difficult case, as we should solve a famous equation, the equation of Kompaneet. The result is still a power law, whose spectral index is approximately given by

$$\alpha = -\frac{3}{2} + \sqrt{\frac{9}{4} + \frac{4}{y}} \tag{5.75}$$

The case $\tau_{\rm T} \gg 1$: saturation

In this case the interaction between photons and matters is so intense that they go to equilibrium, and they will have the same temperature. But instead of a black–body, the resulting photon spectrum has a Wien shape. This is because the photons are conserved (if other scattering processes such as induced Compton or two–photon scattering are important, then one recovers a black–body, because these processes *do not* conserve photons). The Wien spectrum has the slope:

$$I(x) \propto x^3 e^{-x/\Theta} \tag{5.76}$$

At low frequencies this is *harder* than a black–body.

The case $\tau_{\rm T} > 1$, $y > 1$: quasi–saturation

Suppose that in a source characterized by a large $\tau_{\rm T}$ the source of soft photons is spread throughout the source. In this case the photons produced close to the surface, in a skin of optical depth $\tau_{\rm T} = 1$, leave the source without doing any scattering (note that having the source of seed photons concentrated at the center is a different case). The remaining fraction, $1 - 1/\tau_{\rm T}$, i.e. *almost* all photons, remains inside. This can be said for each scattering order. This is illustrated in Fig. 5.11, where $\tau_{\rm T}$ corresponds to the ratio between the flux of photon inside the source at a given frequency and the flux of photons that escape. If I start with 100 photons, only 1 – say – escape, and the other 99 remain inside, and do the first scattering. After it, only one escape, and the other 98 remain inside, and so on, until the typical photon and electron energies are equal, and the photon therefore stays around with the same final frequency until it is its turn to escape. This "accumulation" of photons at $x \sim 3\Theta$ gives the Wien bump. Note that since at any scattering order only a fixed number of photons escape, always the same, then the spectrum in this region will always have $\alpha = 0$. This is a "saturated" index, i.e. one obtains always zero even when changing $\tau_{\rm T}$ or Θ. What indeed changes, by increasing $\tau_{\rm T}$, is that i) the flux characterized by x^0 decreases, ii) the Wien peak will start to dominate earlier (at lower frequencies), while nothing happens to the flux of the Wien peak (it stays there). Increasing still $\tau_{\rm T}$ we fall in the previous case (equilibrium, meaning only the Wien spectrum without the x^0 part).

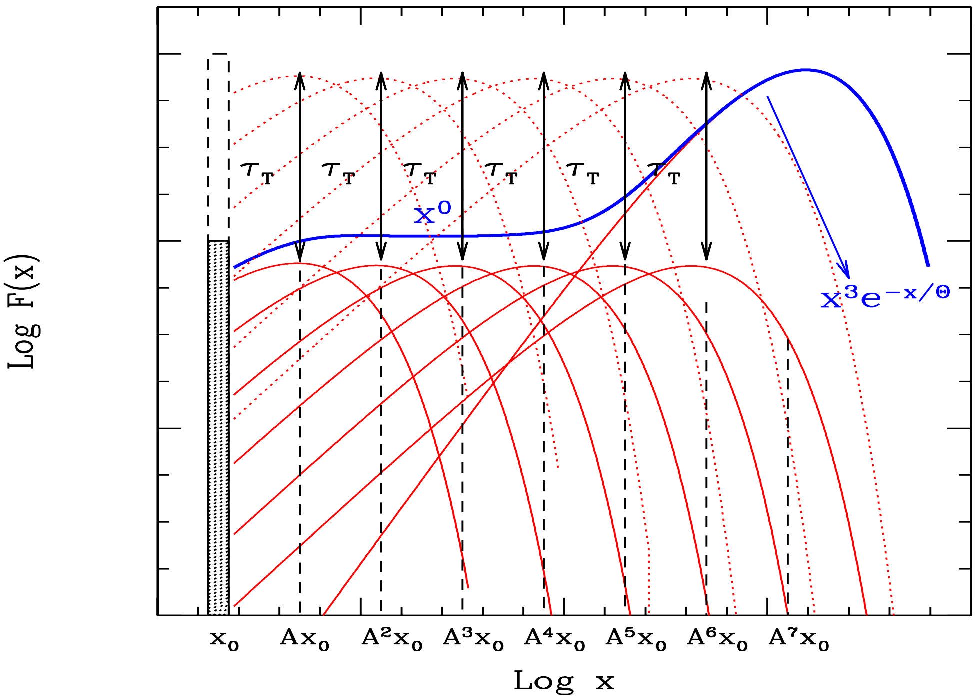

Figure 5.11: Multiple Compton scatterings when $\tau_T > 1$ and $y \gg 1$. For the first scattering orders, *nearly all* photons are scattered: only a fraction $1/\tau_T$ can escape. Therefore the number of photons escaping at each scattering order is the same. This is the reason of the flat part, where $F(x) \propto x^0$. When the photon frequency is of the order of Θ, photons and electrons are in equilibrium, and even if only a small fraction of photons can escape at each scattering order, they do not change frequency any longer, and therefore they form the *Wien bump*, with the slope $F(x) \propto x^3 e^{-x/\Theta}$. If we increase τ_T, the flux with slope x^0 decreases, while the Wien bump stays the same.

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Chapter 6

Synchrotron Self–Compton

Consider a population of relativistic electrons in a magnetized region. They will produce synchrotron radiation, and therefore they will fill the region with photons. These synchrotron photons will have some probability to interact again with the electrons, by the Inverse Compton process. Since the electron "work twice" (first making synchrotron radiation, then scattering it at higher energies) this particular kind of process is called synchrotron self–Compton, or SSC for short.

6.1 SSC emissivity

The importance of the scattering will of course be high if the densities of electrons and photons are large. If the electron distribution is a power law [$N(\gamma) = K\gamma^{-p}$], then we expect that the SSC flux will be $\propto K^2$, i.e. *quadratic* in the electron density.

We should remember Eq. 5.54, and, instead of a generic $U_{\rm r}(\nu)$, we should substitute the appropriate expression for the specific synchrotron radiation energy density. We will then set:

$$U_{\rm s}(\nu) \;=\; \frac{3R}{4c}\,\frac{L_{\rm s}(\nu)}{V} \;=\; 4\pi\,\frac{3R}{4c}\,\epsilon_{\rm s}(\nu) \tag{6.1}$$

where $3R/(4c)$ is the average photon source–crossing time, and V is the volume of the source. Now a simple trick: we write the specific synchrotron emissivity as

$$\epsilon_{\rm s}(\nu) \;=\; \epsilon_{\rm s,0}\,\nu^{-\alpha} \tag{6.2}$$

Remember: the α appearing here *is the same* index in Eq. 5.54. Substituting the above equations into Eq. 5.54 we have

$$\epsilon_{\rm ssc}(\nu_{\rm c}) \;=\; \frac{(4/3)^{\alpha-1}}{2}\,\tau_{\rm c}\,\epsilon_{\rm s,0}\nu_{\rm c}^{-\alpha}\int_{\nu_{\rm min}}^{\nu_{\rm max}}\frac{d\nu}{\nu} \tag{6.3}$$

As you can see, $\epsilon_{s,0}\nu_c^{-\alpha} = \epsilon_s(\nu_c)$ is nothing else than the specific synchrotron emissivity calculated at the (Compton) frequency ν_c. Furthermore, the integral gives a logarithmic term, that we will call $\ln\Lambda$. We finally have:

$$\epsilon_{ssc}(\nu_c) = \frac{(4/3)^{\alpha-1}}{2}\tau_c\,\epsilon_s(\nu_c)\ln\Lambda \tag{6.4}$$

In this form the ratio between the synchrotron and the SSC flux is clear, it is $[(4/3)^{\alpha-1}/2]\tau_c\ln\Lambda \sim \tau_c\ln\Lambda$. It is also clear that since $\tau_c \equiv \sigma_T RK$ and $\epsilon_s(\nu_c) \propto KB^{1+\alpha}$, then, as we have guessed, the SSC emissivity $\epsilon_{ssc}(\nu_c) \propto K^2$ (i.e. electrons work twice). Fig. 6.1 summarizes the main results.

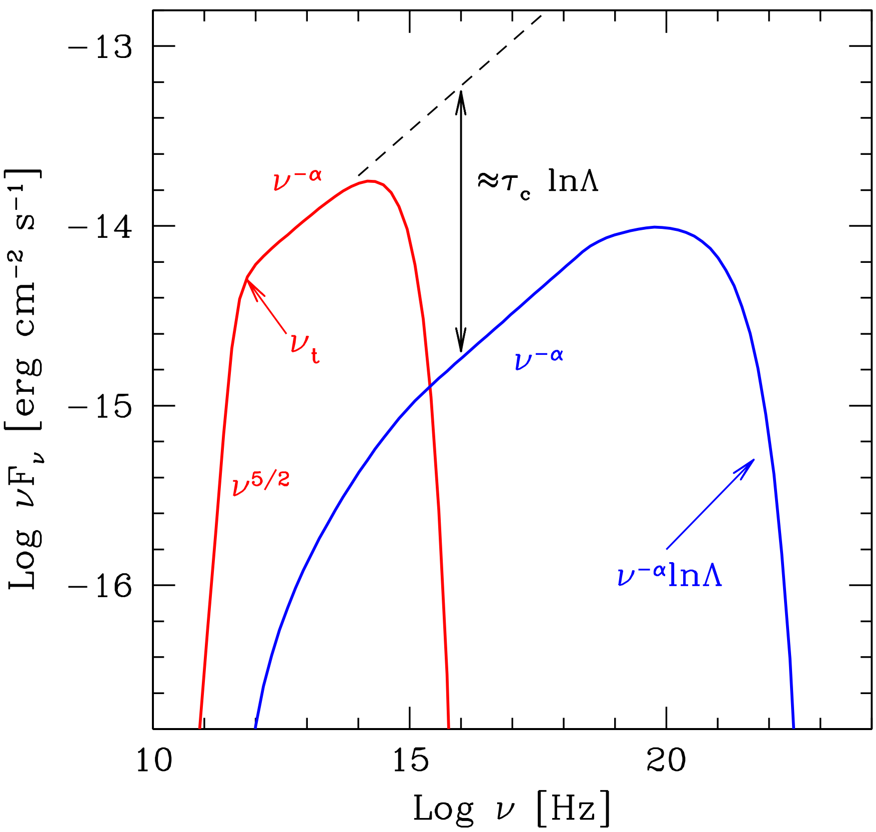

Figure 6.1: Typical example of SSC spectrum, shown in the νF_ν vs ν representation. The spectral indices instead correspond to the $F_\nu \propto \nu^{-\alpha}$ convention.

6.2 Diagnostic

If we are confident that the spectrum of a particular source is indeed given by the SSC process, then we can use our theory to estimate a number of physical parameters. We have already stated (see Eq. 4.32) that observations of the synchrotron spectrum in its self–absorbed part can yield the value of the magnetic field if we also know the angular radius of the source (if it is resolved). Observations in the thin part can then give us the product $RK \equiv \tau_{\rm c}/\sigma_{\rm T}$ (see Eq. 4.28). But $\tau_{\rm c}$ is exactly what we need to predict the high energy flux produced by the SSC process. Note that if the source is resolved (i.e. we know θ_s) we can get these information even without knowing the distance of the source. To summarize:

$$\begin{aligned} F^{\rm syn}_{\rm thick}(\nu) &\propto \theta_s^2 \frac{\nu^{5/2}}{B^{1/2}} & \to \text{ get } B \\ F^{\rm syn}_{\rm thin}(\nu) &\propto \theta_s^2 RKB^{1+\alpha}\nu^{-\alpha} & \to \text{ get } \tau_{\rm c} = RK/\sigma_{\rm T} \end{aligned} \qquad (6.5)$$

There is an even simpler case, which for reasons outlined below, is the most common case employed when studying radio–loud AGNs. In fact, if you imagine to observe the source at the self absorption frequency $\nu_{\rm t}$, then you are both observing the thick and the thin flux at the same time. Then, let us call the flux at $\nu_{\rm t}$ simply $F_{\rm t}$. We can then re-write the equation above:

$$\begin{aligned} B &\propto \frac{\theta_s^4 \nu_{\rm t}^5}{F_{\rm t}^2} \\ \tau_{\rm c} &\propto \frac{F_{\rm t}\nu_{\rm t}^\alpha}{\theta_s^2 B^{1+\alpha}} \\ F_{\rm ssc}(\nu_{\rm c}) &\propto \tau_{\rm c} F_{syn}(\nu_{\rm c}) \propto \tau_{\rm c}^2 B^{1+\alpha}\nu_{\rm c}^{-\alpha} \\ &\propto F_{\rm t}^{2(2+\alpha)}\nu_{\rm t}^{-(5+3\alpha)}\theta_s^{-2(3+2\alpha)}\nu_{\rm c}^{-\alpha} \end{aligned} \qquad (6.6)$$

Once again: on the basis of a few observations of only the synchrotron flux, we can calculate what should be the SSC flux at the frequency $\nu_{\rm c}$. Note the rather strong dependencies, particularly for θ_s, in the sense that the more compact the source is, the larger the SSC flux.

If it happens that we do observe the source at high frequencies, where we expect that the SSC flux dominates, then we can check if our model works. Does it? For the strongest radio–loud sources, almost never. The disagreement between the predicted and the observed flux is really severe, we are talking of several orders of magnitude. Then either we are completely wrong about the model, or we miss some fundamental ingredient. We go for the second option, since, after all, we do not find any mistake in our theory.

The missing ingredient is relativistic bulk motion. If the source is moving towards us at relativistic velocities, we observe an enhanced flux and blueshifted frequencies. Not accounting for it, our estimates of the magnetic field and particles densities are wrong, in the sense that the B field is smaller than the real one, and the particle densities are much greater (for smaller B we need more particle to produce the same synchrotron flux). So we repeat the entire procedure, but this time assuming that $F(\nu) = \delta^{3+\alpha} F'(\nu)$, where $\delta = 1/[\Gamma(1-\beta\cos\theta)]$ is the Doppler factor and $F'(\nu)$ is the flux received by a comoving observer at the same frequency ν. Then

$$\begin{aligned} F^{\rm syn}_{\rm thick}(\nu) &\propto \theta_s^2 \frac{\nu^{5/2}}{B^{1/2}} \delta^{1/2} \\ F^{\rm syn}_{\rm thin}(\nu) &\propto \theta_s^2 RKB^{1+\alpha}\nu^{-\alpha}\delta^{3+\alpha} \end{aligned} \tag{6.7}$$

The predicted SSC flux then becomes

$$F_{\rm ssc}(\nu_{\rm c}) \propto F_{\rm t}^{2(2+\alpha)} \nu_{\rm t}^{-(5+3\alpha)} \theta_s^{-2(3+2\alpha)} \nu_{\rm c}^{-\alpha} \delta^{-2(2+\alpha)} \tag{6.8}$$

If we now compare the predicted with the observe SSC flux, we can estimate δ. And indeed this is one of the most powerful δ–estimators, even if it is not the only one.

6.3 Why it works

We have insisted on the importance of observing the synchrotron flux both in the self–absorbed and in the thin regime, to get B and $\tau_{\rm c}$. But the self–absorbed part of the synchrotron spectrum, the one $\propto \nu^{5/2}$ is very rarely observed in general, and never in radio–loud AGNs. So, where is the trick? It is the following. In radio–loud AGN the synchrotron emission, at radio frequencies, comes partly from the radio lobes (extended structures, hundreds of kpc in size, very relaxed, unbeamed, and usually self–absorbing at very small frequencies) and from the jet. The emission from the latter is beamed, and it is the superposition of the fluxes produced in several regions: the most compact ones (closer to the central engine) self–absorb at high radio frequencies (say, at 100 GHz), and the bigger they are, the smaller their self–absorbed frequency. But what is extraordinary about these jets is that the peak flux of each component (i.e. the flux at the self–absorption frequency) is approximately constant (in the past, this phenomenon was called *cosmic conspiracy*). Therefore, when we sum up all the components, we have a flat radio spectrum, as illustrated by Fig. 6.2.

Of course the emission components of the jet, to behave in such a coherent way, must have an electron density and a magnetic field that decrease with the distance from the central engine in an appropriate way. There is a family of solutions, but the most appealing is certainly $B(R) \propto R^{-1}$ and $K(R) \propto R^{-2}$. It is appealing because it corresponds to conservation of the total number of particles, conservation of the bulk power carried by them (if Γ does not change) and conservation of the Poynting flux (i.e. the power carried by the magnetic field).

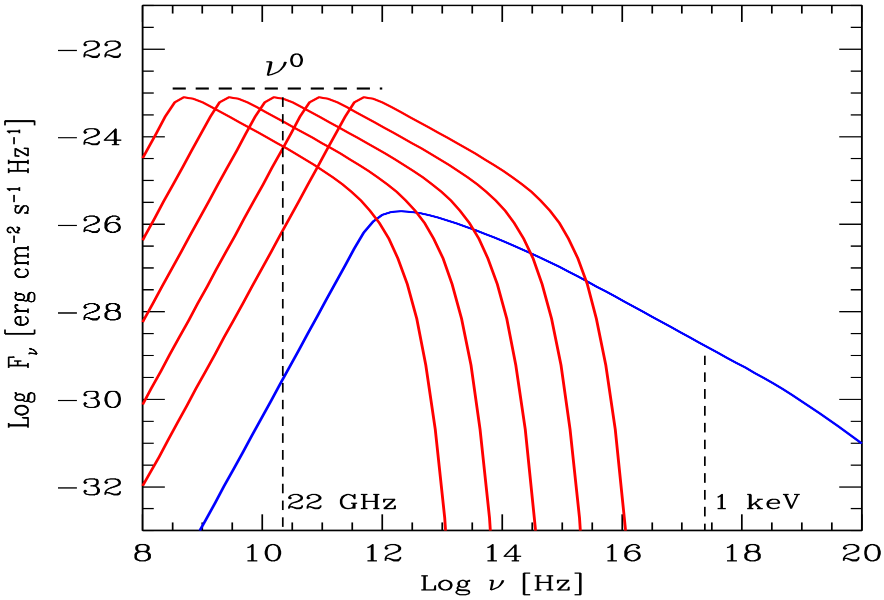

Figure 6.2: Typical example of the composite spectrum of a flat spectrum quasars (FSRQ) shown in the F_ν vs ν representation, to better see the flat spectrum in the radio. The reason of the flat spectrum is that different parts of the jet contributes at different frequencies, but in a coherent way. The blue line is the SSC spectrum. Suppose to observe, with the VLBI, at 22 GHz: in this framework we will always observe the jet component peaking at this frequency. So you automatically observe at the self–absorption frequency of that component (for which you measure the angular size).

To our aims, the fact that the jet has many radio emission sites self–absorbing at different frequency is of great help. In fact suppose to observe a jet with the VLBI, at one frequency, say 22 GHz. There is a great chance to observe the emission zone which is contributing the most to that frequency, i.e. the one which is self–absorbing at 22 GHz. At the same time you measure the size. Then, suppose to know the X–ray flux of the source. It will be the X–ray flux not only of that component you see with the VLBI, but an integrated flux from all the inner jet (with X–ray instruments the maximum angular resolution is about 1 arcsec, as in optical). But nevertheless you know that your radio blob cannot exceed the measured, total, X–ray flux. Therefore you can put a limit on δ (including constants):

$$\delta \quad > \quad (0.08\alpha + 0.14)\,(1+z)\left(\frac{F_{\rm t}}{\rm Jy}\right)\left(\frac{F_x}{\rm Jy}\right)^{-\frac{1}{2(2+\alpha)}}\left(\frac{\nu_x}{1\ {\rm keV}}\right)^{-\frac{\alpha}{2(2+\alpha)}}$$
$$\times \quad \left(\frac{\nu_{\rm t}}{5\ {\rm GHz}}\right)^{-\frac{5+3\alpha}{2(2+\alpha)}}\left(\frac{2\theta_s}{\rm m.a.s.}\right)^{-\frac{3+2\alpha}{2+\alpha}}\left[\ln\left(\frac{\nu_{\rm s,max}}{\nu_{\rm t}}\right)\right]^{\frac{1}{2(2+\alpha)}} \qquad (6.9)$$

For some sources you would find $\delta > 10$ or 20, i.e. rather large values.